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RESEARCH TITLE

Coupled Nonlinear Schrödinger Equations for Pulse Interactions in Optical Fibers: Self- and Cross-Phase Modulation Effects

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Abstract

In this paper, we explore the pulse interactions in optical fibers governed by the Coupled Nonlinear Schrödinger Equations (CNLSEs), considering both self- and cross-phase modulation effects. These equations play a crucial role in understanding the dynamics of pulse propagation in nonlinear optical media, especially in scenarios involving multiple interacting pulses. To investigate this, we apply the (G'/G)-Expansion Method, a powerful analytical technique for deriving exact solutions to nonlinear differential equations. By employing this method, we derive a family of explicit traveling wave solutions, expressed in terms of hyperbolic, trigonometric, and rational functions. These solutions provide valuable insight into the physical phenomena associated with the interaction of pulses in optical fibers, such as the modulation and stability of pulse envelopes. The results demonstrate the effectiveness of the (G'/G)-Expansion Method in solving complex coupled systems and contribute to a deeper understanding of nonlinear optical effects, with potential applications in optical communication systems and pulse shaping technologies. Numerical simulations are presented to illustrate the behavior of the obtained solutions under different parameter settings, highlighting the significance of self- and cross-phase modulation in pulse dynamics.

Key Words: Coupled Nonlinear Schrödinger Equations (CNLSEs); Pulse interactions; Self- and cross-phase modulation; The (G'/G)-expansion method; Optical fibers.

1. Introduction

Nonlinear optical phenomena have gained considerable attention in recent decades due to their fundamental role in the propagation and interaction of light pulses in optical fibers. A key model used to describe these phenomena is the Nonlinear Schrödinger Equation (NLSE), which governs the evolution of optical pulses under the effects of group velocity dispersion and Kerr nonlinearity. In systems where multiple pulses or channels propagate, the Coupled Nonlinear Schrödinger Equations (CNLSEs) are required to account for the interaction between different modes or wavelengths, making them indispensable in understanding multi-pulse dynamics in fiber-optic communications and pulse-shaping technologies [1-12].

Self-phase modulation (SPM) and cross-phase modulation (XPM) are two pivotal effects arising from the nonlinear refractive index of optical fibers. SPM describes the frequency chirping of an optical pulse caused by its own intensity, while XPM represents the influence of one pulse's intensity on the phase of another co-propagating pulse. These effects significantly impact the stability, shape, and interaction of optical pulses, especially in high-bit-rate fiber-optic systems where signal distortion must be minimized [13-24]. The intricate nature of these interactions calls for advanced analytical techniques to derive exact solutions for the CNLSEs, allowing deeper insight into the underlying physics.

The (G'/G)-Expansion Method has emerged as a powerful tool for solving nonlinear partial differential equations (PDEs) such as the NLSE and its coupled counterparts. This method facilitates the construction of exact traveling wave solutions in terms of hyperbolic, trigonometric, and rational functions, making it an effective approach for studying pulse interactions in nonlinear optical systems [25-30]. In this work, we apply the (G'/G)-Expansion Method to the CNLSEs to derive explicit solutions that elucidate the role of SPM and XPM in pulse evolution. These solutions are not only useful for understanding the dynamics of optical pulses but also for improving the design of fiber-optic communication systems, where precise control of pulse propagation is crucial.

2. The (G'/G)-Expansion Method

Suppose we have the following nonlinear partial differential equation:

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \cdots) = 0. \qquad (1)$$

Where $u = u(x,t)$ is an unknown function, P is a polynomial in u and its partial derivatives that involves the highest-order derivatives and the nonlinear terms.

Main Steps of the (G'/G)-expansion Method:

• **Step 1:** The traveling wave variable

$$u(x,t) = u(\xi), \quad \xi = x - Vt \qquad (2)$$

Transforms Eq. (1) into an ordinary differential equation (ODE):

$$Q(u, u', u'', u''', \cdots) = 0, \qquad (3)$$

Where prime denotes the derivative with respect to ξ.

• **Step 2:** If Eq. (3) is integrable, integrate it term by term to yield constant(s) of integration.

• **Step 3:** Assume the solution can be expressed as a polynomial in $\left(\frac{G'}{G}\right)$ as follows:

$$u(\xi) = \sum_{i=0}^{n} a_i \left(\frac{G'}{G}\right)^i, \quad a_n \neq 0 \tag{4}$$

Where $G = G(\xi)$ meets the generalized Riccati equation:

$$G' = r + pG + qG^2 \tag{5}$$

Where $a_i, (i = 1,2,3,\dots,n), p, q$ and r are constants to be determined later.

The generalized Riccati Eq. (5) has the following twenty seven solutions [49].

Family 1. When $p^2 - 4qr < 0$ and $pq \neq 0$ (or $qr \neq 0$), the solutions of Eq. (5) are,

$$G_1 = \frac{1}{2q}\left[-p + \sqrt{4qr - p^2}\tan\left(\frac{1}{2}\sqrt{4qr - p^2}\xi\right)\right], \quad G_2 = -\frac{1}{2q}\left[p + \sqrt{4qr - p^2}\cot\left(\frac{1}{2}\sqrt{4qr - p^2}\xi\right)\right],$$

$$G_3 = \frac{1}{2q}\left[p + \sqrt{4qr - p^2}\left(\tan\left(\sqrt{4qr - p^2}\xi\right) \pm \sec\left(\sqrt{4qr - p^2}\xi\right)\right)\right],$$

$$G_4 = -\frac{1}{2q}\left[p + \sqrt{4qr - p^2}\left(\cot\left(\sqrt{4qr - p^2}\xi\right) \pm \csc\left(\sqrt{4qr - p^2}\xi\right)\right)\right],$$

$$G_5 = \frac{1}{4q}\left[-2p + \sqrt{4qr - p^2}\left(\tan\left(\frac{1}{4}\sqrt{4qr - p^2}\xi\right) - \cot\left(\frac{1}{4}\sqrt{4qr - p^2}\xi\right)\right)\right],$$

$$G_6 = \frac{1}{2q}\left[-p + \frac{\sqrt{(A^2-B^2)(4qr-p^2)} - A\sqrt{4qr-p^2}\cos\left(\sqrt{4qr-p^2}\xi\right)}{A\sin\left(\sqrt{4qr-p^2}\xi\right)+B}\right],$$

$$G_7 = \frac{1}{2q}\left[-p + \frac{\sqrt{(A^2-B^2)(4qr-p^2)} + A\sqrt{4qr-p^2}\cos\left(\sqrt{4qr-p^2}\xi\right)}{A\sin\left(\sqrt{4qr-p^2}\xi\right)+B}\right],$$

where A and B are two non-zero real constants and satisfies the condition $A^2 - B^2 > 0$.

$$G_8 = \frac{-2r\cos\left(\frac{1}{2}\sqrt{4qr-p^2}\xi\right)}{\sqrt{4qr-p^2}\sin\left(\frac{1}{2}\sqrt{4qr-p^2}\xi\right)+p\cos\left(\frac{1}{2}\sqrt{4qr-p^2}\xi\right)},$$

$$G_9 = \frac{2r\sin\left(\frac{1}{2}\sqrt{4qr-p^2}\xi\right)}{-p\sin\left(\frac{1}{2}\sqrt{4qr-p^2}\xi\right)+\sqrt{(4qr-p^2)}\cos\left(\frac{1}{2}\sqrt{4qr-p^2}\xi\right)},$$

$$G_{10} = \frac{-2r\cos\left(\sqrt{4qr-p^2}\xi\right)}{\sqrt{(4qr-p^2)}\sin\left(\sqrt{4qr-p^2}\xi\right)+p\cos\left(\sqrt{4qr-p^2}\xi\right)\pm\sqrt{(4qr-p^2)}},$$

$$G_{11} = \frac{2r\sin\left(\sqrt{4qr-p^2}\xi\right)}{-p\sin\left(\sqrt{4qr-p^2}\xi\right)+\sqrt{(4qr-p^2)}\cos\left(\sqrt{4qr-p^2}\xi\right)\pm\sqrt{(4qr-p^2)}},$$

$$G_{12} = \frac{4r\sin\left(\frac{1}{4}\sqrt{4qr-p^2}\xi\right)\cos\left(\frac{1}{4}\sqrt{4qr-p^2}\xi\right)}{-2p\sin\left(\frac{1}{4}\sqrt{4qr-p^2}\xi\right)\cos\left(\frac{1}{4}\sqrt{4qr-p^2}\xi\right)+2\sqrt{(4qr-p^2)}\cos^2\left(\frac{1}{4}\sqrt{4qr-p^2}\xi\right)-\sqrt{(4qr-p^2)}}.$$

Family 2. When $p^2 - 4qr > 0$ and $pq \neq 0$ (or $qr \neq 0$), the solutions of Eq. (5) are,

$$G_{13} = -\frac{1}{2q}\left[p + \sqrt{p^2 - 4qr}\tanh\left(\frac{1}{2}\sqrt{p^2 - 4qr}\xi\right)\right],$$

$$G_{14} = -\frac{1}{2q}\left[p + \sqrt{p^2 - 4qr}\,coth\left(\frac{1}{2}\sqrt{p^2 - 4qr}\xi\right)\right],$$

$$G_{15} = -\frac{1}{2q}\left[p + \sqrt{p^2 - 4qr}\left(\tanh\left(\sqrt{p^2 - 4qr}\xi\right) \pm i\,sech\left(\sqrt{p^2 - 4qr}\xi\right)\right)\right],$$

$$G_{16} = -\frac{1}{2q}\left[p + \sqrt{p^2 - 4qr}\left(coth\left(\sqrt{p^2 - 4qr}\xi\right) \pm csch\left(\sqrt{p^2 - 4qr}\xi\right)\right)\right],$$

$$G_{17} = -\frac{1}{4q}\left[2p + \sqrt{p^2 - 4qr}\left(\tanh\left(\frac{1}{4}\sqrt{p^2 - 4qr}\xi\right) + coth\left(\frac{1}{4}\sqrt{p^2 - 4qr}\xi\right)\right)\right],$$

$$G_{18} = \frac{1}{2q}\left[-p + \frac{\sqrt{(A^2+B^2)(p^2-4qr)} - A\sqrt{p^2-4qr}\cosh\left(\sqrt{p^2-4qr}\xi\right)}{A\sinh\left(\sqrt{p^2-4qr}\xi\right)+B}\right],$$

$$G_{19} = \frac{1}{2q}\left[-p - \frac{\sqrt{(B^2-A^2)(p^2-4qr)} + A\sqrt{p^2-4qr}\cosh\left(\sqrt{p^2-4qr}\xi\right)}{A\sinh\left(\sqrt{p^2-4qr}\xi\right)+B}\right],$$

where A and B are two non-zero real constants and satisfies the condition $B^2 - A^2 > 0$.

$$G_{20} = \frac{2r\cosh\left(\frac{1}{2}\sqrt{p^2-4qr}\xi\right)}{\sqrt{p^2-4qr}\sinh\left(\frac{1}{2}\sqrt{p^2-4qr}\xi\right) - p\cosh\left(\frac{1}{2}\sqrt{p^2-4qr}\xi\right)},$$

$$G_{21} = \frac{2r\sinh\left(\frac{1}{2}\sqrt{p^2-4qr}\xi\right)}{\sqrt{p^2-4qr}r\cosh\left(\frac{1}{2}\sqrt{p^2-4qr}\xi\right) - p\sinh\left(\frac{1}{2}\sqrt{p^2-4qr}\xi\right)},$$

$$G_{22} = \frac{2r\cosh\left(\sqrt{p^2-4qr}\xi\right)}{\sqrt{p^2-4qr}\sinh\left(\sqrt{p^2-4qr}\xi\right) - p\cosh\left(\sqrt{p^2-4qr}\xi\right) \pm i\sqrt{p^2-4qr}},$$

$$G_{23} = \frac{2r\sinh\left(\sqrt{p^2-4qr}\xi\right)}{-p\sinh\left(\sqrt{p^2-4qr}\xi\right) + \sqrt{p^2-4qr}\cosh\left(\sqrt{p^2-4qr}\xi\right) \pm \sqrt{p^2-4qr}},$$

$$G_{24} = \frac{4r\sinh\left(\frac{1}{4}\sqrt{p^2-4qr}\xi\right)\cosh\left(\frac{1}{4}\sqrt{p^2-4qr}\xi\right)}{-2p\sinh\left(\frac{1}{4}\sqrt{p^2-4qr}\xi\right)\cosh\left(\frac{1}{4}\sqrt{p^2-4qr}\xi\right) + 2\sqrt{p^2-4qr}\cosh^2\left(\frac{1}{4}\sqrt{p^2-4qr}\xi\right) - \sqrt{p^2-4qr}}.$$

Family 3. When $r = 0$ and $pq \neq 0$, the solutions of Eq. (5) are,

$$G_{25} = \frac{-pd}{q[d + \cosh(p\xi) - \sinh(p\xi)]},$$

$$G_{26} = -\frac{p[\cosh(p\xi) + \sinh(p\xi)]}{q[d + \cosh(p\xi) + \sinh(p\xi)]},$$

where d is an arbitrary constant.

Family 4. When $q \neq 0$ and $r = p = 0$, the solution of Eq. (5) is,

$$G_{27} = -\frac{1}{q\xi + d_1}$$

where d_1 is an arbitrary constant.

• **Step 4:** In Eq. (4), n is a positive integer obtained by balancing the highest-order nonlinear term(s) with the linear term(s) of the highest order from Eq. (3).

• **Step 5:** Substitute Eq. (4) into Eq. (3) and use Eq. (5) to obtain polynomials in G^i and G^{-i}. By setting each coefficient to zero, we derive a set of algebraic equations for a_i, p, q, r, V and any constants of integration.

3. Application of The (G'/G)-Expansion Method to The Coupled Nonlinear Schrödinger Equations for Pulse Interactions in Optical Fibers: Self- and Cross-Phase Modulation Effects.

The propagation of optical pulses in fibers is fundamentally influenced by nonlinear effects such as self-phase modulation (SPM) and cross-phase modulation (XPM). These effects, which arise from the nonlinear refractive index of the fiber, play a critical role in shaping pulse interactions, particularly in systems where multiple pulses or channels co-propagate. The Coupled Nonlinear Schrödinger Equations (CNLSEs) provide a mathematical framework to describe these interactions. In this study, we apply the (G'/G)-Expansion Method to the CNLSEs to derive exact solutions that capture the dynamics of pulse evolution under SPM and XPM effects. These solutions offer valuable insights for the design and optimization of fiber-optic communication systems.

The model for Self- and Cross-Phase Modulation Effects can be represented by the following equations:

$$u_{xt} - Vu_{yy} - \mu u_t + \alpha u - \beta|u|^2 u - \gamma|v|^2 u = 0$$

$$v_{yt} + Vv_{xx} - \mu v_t + \alpha v - \beta|v|^2 v - \gamma|u|^2 v = 0$$

Explanation of Variables and Terms:

1. $u(x,y,t)$ and $u(x,y,t)$: These represent the complex envelopes of the two interacting optical pulses (or modes). Each of these functions describes the amplitude and phase of the light waves in two different modes, channels, or polarizations within the optical fiber. They are functions of both spatial coordinates (x,y) and time (t).

2. u_{xt} and v_{yt}: These mixed partial derivatives describe the coupling between spatial and temporal evolution for the two pulses. It reflects how the propagation of the pulse changes with both position and time.

3. Vu_{yy} and Vv_{xx}: These terms represent group velocity dispersion (GVD) in the optical fiber. The factor V is related to the dispersion parameter, which governs how the pulse spreads over time due to different frequency components traveling at different speeds. The subscripts (yy) and (xx) refer to the second spatial derivatives, showing that the pulse spreads in space due to dispersion effects.

4. μu_t and μv_t: These are dissipative terms that describe the effects of attenuation or loss in the fiber, characterized by the parameter (μ). As the optical pulse travels, its intensity may decrease due to absorption or scattering in the fiber material.

5. αu and αv: These terms represent linear effects on the propagation of the pulses, such as linear amplification or refractive index effects. The parameter (α) governs the strength of these linear effects on the wave envelope.

6. $-\beta|u|^2 u$ and $-\beta|v|^2 v$: These are the self-phase modulation (SPM) terms. SPM arises because the refractive index of the fiber depends on the intensity of the light. As a result, each pulse modifies its own phase due to its intensity. The parameter (β) governs the strength of this nonlinear self-interaction.

− $|u|^2$ represents the intensity of the pulse (u), while $|v|^2$ represents the intensity of the pulse (v).
− The cubic term $|u|^2 u$ in the first equation and $|v|^2 v$ in the second equation means that each pulse undergoes a self-induced phase shift due to its own intensity.

7. $-\gamma|v|^2 u$ and $-\gamma|u|^2 v$: These are the cross-phase modulation (XPM) terms. XPM occurs when the intensity of one pulse affects the phase of another co-propagating pulse. The parameter (γ) dictates the strength of this cross-interaction between the two pulses.

− In the first equation, the term $\gamma|v|^2 u$ indicates that the intensity of the pulse (v) influences the phase of (u).
− Similarly, in the second equation, $\gamma|u|^2 v$ shows that the intensity of (u) affects the phase of (v).

Physical Interpretation:

− Self-Phase Modulation (SPM): SPM refers to the effect where a pulse changes its own phase due to its intensity, leading to frequency chirping (a shift in the frequency components of the pulse). This effect can lead to pulse broadening as it propagates through the fiber.

− Cross-Phase Modulation (XPM): XPM describes how the intensity of one pulse affects the phase of another pulse. In a multi-channel or multi-mode optical system, XPM plays a crucial role, especially in wavelength-division multiplexing (WDM) systems, where signals at different wavelengths interact nonlinearly. XPM can lead to signal distortion if not properly managed.

Coupled Nature of the Equations:
The two equations are coupled due to the presence of the XPM terms $|v|^2u$ and $|u|^2v$. This coupling reflects the physical interaction between the two pulses. If only one pulse exists (e.g., $(v = 0)$), the first equation reduces to a simpler form, governing the self-phase modulation for that single pulse. However, when both pulses are present, their intensities mutually influence each other's propagation through the XPM terms.

Mathematical preliminaries

Assume that

$$u(x,y,t) = w_1(k_1x + k_2y - \tau_1t)e^{i(a_1x+\tau_2t)}$$
$$v(x,y,t) = w_2(ik_1x + k_2y - \tau_1t)e^{i(a_2x+\tau_2t)}$$

Now, using the traveling wave variable (2) in Eq. (1), we have
$-(k_1\tau_1 + k_2^2V)w_1'' + \tau_1\mu w_1' + (\alpha - a_1\tau_2)w_1 - \beta w_1^2 - \gamma w_2^2 + i[(k_1\tau_2 - a_1\tau_1)w_1' - \tau_2\mu w_1] = 0$ (6)
and
$-(k_2\tau_1 + k_1^2V)w_2'' + \tau_1\mu w_2' + (\alpha - a_2\tau_2)w_2 - \beta w_2^2 - \gamma w_1^2 + i[(k_2\tau_2 - a_2\tau_1)w_2' - \tau_2\mu w_2] = 0$ (7)

equation (6) and (7) can be gathered as

$$-(k_j\tau_1 + k_L^2V)w_j'' + \tau_1\mu w_j' + (\alpha - a_j\tau_2)w_j - \beta w_j^2 - \gamma w_L^2 + i[(k_j\tau_2 - a_j\tau_1)w_j' - \tau_2\mu w_j] = 0$$

where $j = 1,2$ and $L = 3 - j$. Setting $w_L = Aw_j$ to obtain

$$-(k_j\tau_1 + k_L^2V)w_j'' + \tau_1\mu w_j' + (\alpha - a_j\tau_2)w_j - (\beta + \gamma A^2)w_j^2 + i[(k_j\tau_2 - a_j\tau_1)w_j' - \tau_2\mu w_j] = 0$$

Splitting into real and imaginary parts we get

Real

$$-(k_j\tau_1 + k_L^2V)w_j'' + \tau_1\mu w_j' + (\alpha - a_j\tau_2)w_j - (\beta + \gamma A^2)w_j^2 = 0$$

Imaginary

$$(k_j\tau_2 - a_j\tau_1)w_j' - \tau_2\mu w_j = 0$$

According to Step 3, the solution of Eq. (3) can be expressed by a polynomial in (G'/G) as follows:

$$w_j = b_0 + b_1\left(\frac{G'}{G}\right) + b_2\left(\frac{G'}{G}\right)^2 + \cdots + b_n\left(\frac{G'}{G}\right)^n, \quad b_n \neq 0$$

where $a_i, (i = 0,1,2,3, \cdots n)$ are constant to be determined and $G = G(\xi)$ satisfies the generalized Riccati Eq. (5). Considering the homogeneous balance between the highest order derivative and the nonlinear terms in Eq. (3), we obtain $n = 2$. Therefore, the solution of Eq. (4) takes the form:

$$w_j = b_0 + b_1\left(\frac{G'}{G}\right) + b_2\left(\frac{G'}{G}\right)^2, \quad b_2 \neq 0$$

Case 1:

$a_j = \alpha + k_l^2\, p^2\, v, \quad k_j = k_j, \quad r = 0, \quad p = p, \quad b_2 = b_2, \quad b_0 = \frac{b_2\, p^2}{6}, \quad b_1 = -b_2\, p, \quad q = q,$
$\tau_1 = \tau_1, \quad \tau_2 = 1, \quad A = \pm\sqrt{-\frac{b_2\,\beta - 6\,k_l^2\,v}{b_2\,\gamma}}$

We notes that Family 1 does not apply here because p^2 can not be less than zero

Family 2.

$$u(x,y,t)_{13} = \left[b_0 + b_1\left(p - \frac{p}{2}\left(1 + \tanh\left(\frac{p}{2}(k_1x + k_2y - \tau_1 t)\right)\right)\right) + b_2\left(p - \frac{p^2}{2}\left(1 + \tanh\left(\frac{p}{2}(k_1x + k_2y - \tau_1 t)\right)\right)\right)^2\right] e^{i(a_1x + \tau_2 t)} \qquad (8)$$

$$v(x,y,t)_{13} = \left[b_0 + b_1\left(p - \frac{p}{2}\left(1 + \tanh\left(\frac{p}{2}(ik_1x + k_2y - \tau_1 t)\right)\right)\right) + b_2\left(p - \frac{p^2}{2}\left(1 + \tanh\left(\frac{p}{2}(ik_1x + k_2y - \tau_1 t)\right)\right)\right)^2\right] e^{i(a_1x + \tau_2 t)} \qquad (9)$$

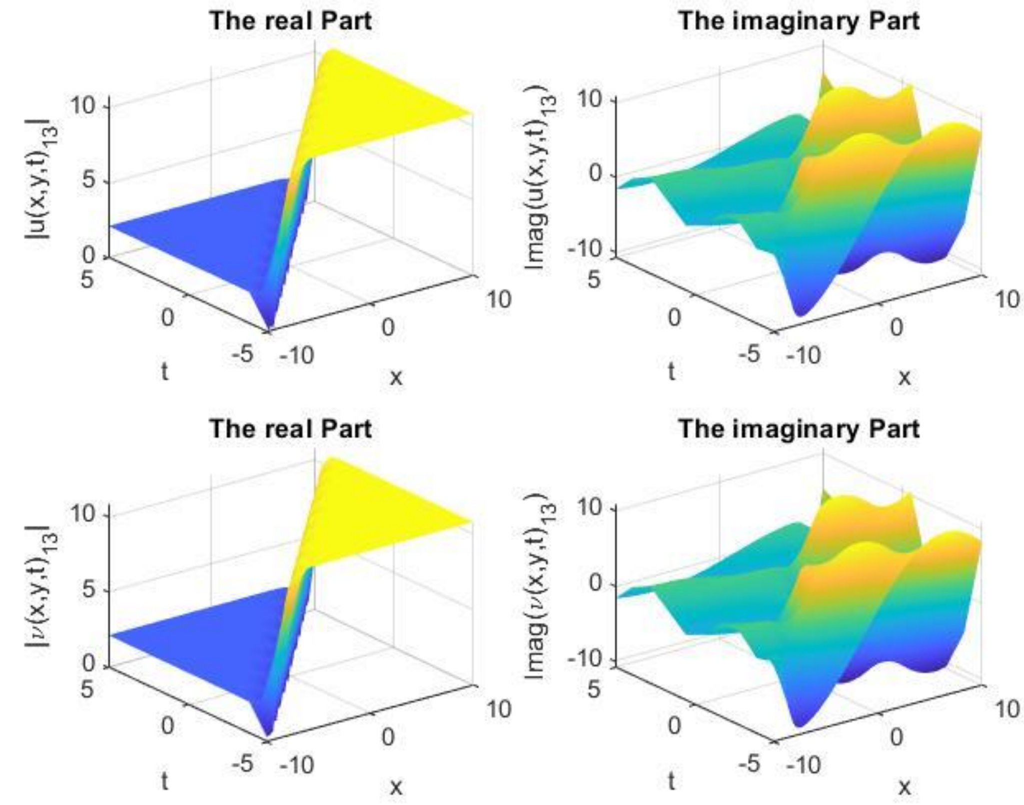

Figure 1: Plotting of the real and the Imaginary parts of $\nu_{13}(x,y,t)$ with $b_0 = 0.01$, $b_1 = -0.19$, $b_2 = 0.3$, $p = 3$, $k_1 = 0.01$, $k_2 = 0.8$, $\tau_1 = 1.5$, $a_1 = 0.2$, $\tau_2 = 1.2$, and $x \in [-10,10]$, $y \in [-10,10]$ and $t \in [-5,5]$.

$$u(x,y,t)_{14} = \left[b_0 + b_1\left(p - \frac{p}{2}\left(1 + coth\left(\frac{p}{2}(k_1x + k_2y - \tau_1t)\right)\right)\right) + b_2\left(p - \frac{p^2}{2}\left(1 + coth\left(\frac{p}{2}(k_1x + k_2y - \tau_1t)\right)\right)\right)^2\right] e^{i(a_1x+\tau_2t)} \quad (10)$$

$$\nu(x,y,t)_{14} = b_0 + b_1\left(p - \frac{p}{2}\left[1 + coth\left(\frac{p}{2}(ik_1x + k_2y - \tau_1t)\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + coth\left(\frac{p}{2}(ik_1x + k_2y - \tau_1t)\right)\right]\right)^2 e^{i(a_1x+\tau_2t)} \quad (11)$$

$$u(x,y,t)_{15} = b_0 + b_1\left(p - \frac{p}{2}\left[1 + \left(\tanh(p(k_1x + k_2y - \tau_1t)) \pm isech(p(k_1x + k_2y - \tau_1t))\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \left(\tanh(p(k_1x + k_2y - \tau_1t)) \pm isech(p(k_1x + k_2y - \tau_1t))\right)\right]\right)^2 e^{i(a_1x+\tau_2t)}$$
(12)

$$\nu(x,y,t)_{15} = b_0 + b_1\left(p - \frac{p}{2}\left[1 + \left(\tanh(p(ik_1x + k_2y - \tau_1t)) \pm isech(p(ik_1x + k_2y - \tau_1t))\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \left(\tanh(p(ik_1x + k_2y - \tau_1t)) \pm isech(p(ik_1x + k_2y - \tau_1t))\right)\right]\right)^2 e^{i(a_1x+\tau_2t)}$$
(13)

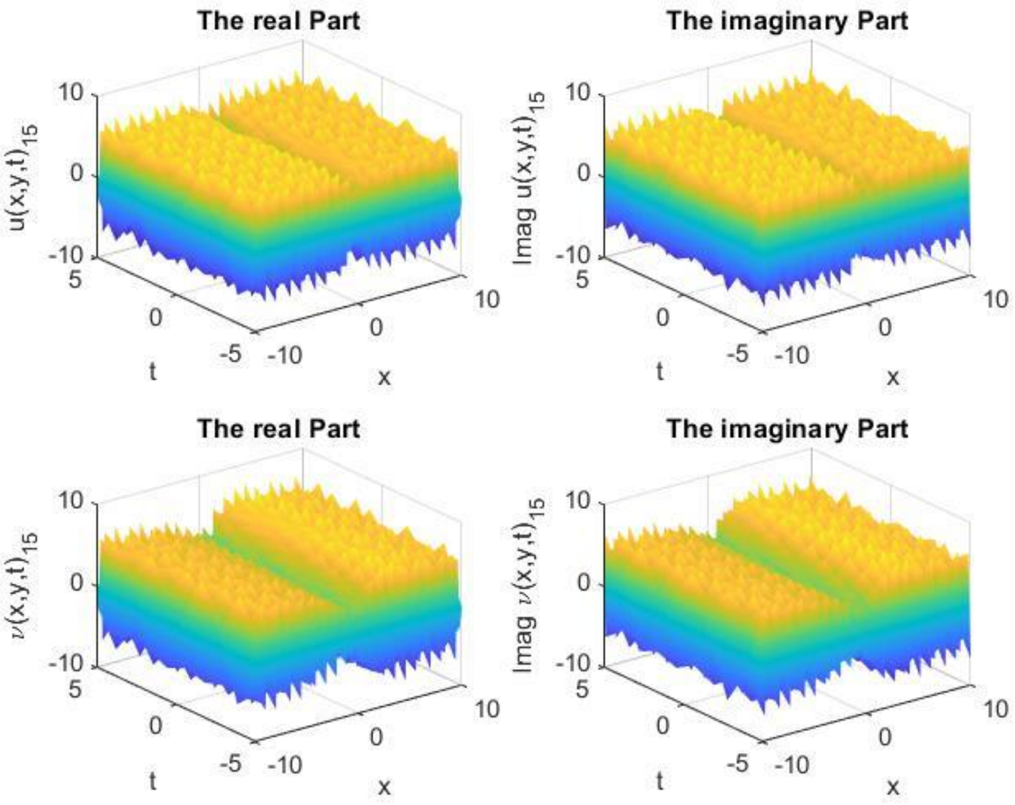

Figure 2: Plotting of the real and the Imaginary parts of $\nu_{15}(x,y,t)$ with $b_0 = 1.5$, $b_1 = 0.19$, $b_2 = 1.3$, $p = 2$, $k_1 = 0.1$, $k_2 = 0.5$, $\tau_1 = 0.01$, $a_1 = 13$, $\tau_2 = 1.2$, and $x \in [-10,10]$, $y \in [-10,10]$ and $t \in [-5,5]$.

$$u(x,y,t)_{16} = b_0 + b_1\left(p - \frac{p}{2}\left[1 + \left(coth(p(k_1x + k_2y - \tau_1 t)) \pm csch(p(k_1x + k_2y - \tau_1 t))\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \left((coth(p(k_1x + k_2y - \tau_1 t)) \pm csch(p(k_1x + k_2y - \tau_1 t))\right)\right]\right)^2 e^{i(a_1x + \tau_2 t)} \quad (14)$$

$$\nu(x,y,t)_{16} = b_0 + b_1\left(p - \frac{p}{2}\left[1 + \left((coth(p(ik_1x + k_2y - \tau_1 t)) \pm csch(p(ik_1x + k_2y - \tau_1 t))\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \left((coth(p(ik_1x + k_2y - \tau_1 t)) \pm csch(p(ik_1x + k_2y - \tau_1 t))\right)\right]\right)^2 e^{i(a_1x + \tau_2 t)} \quad (15)$$

$$u(x,y,t)_{17} = b_0 + b_1\left(p - \frac{p}{4}\left[2 + \left(\tanh\left(\frac{p}{4}(k_1x + k_2y - \tau_1 t)\right) + coth\left(\frac{p}{4}(k_1x + k_2y - \tau_1 t)\right)\right)\right]\right) + b_2\left(p - \frac{p^2}{4}\left[2 + \left(\tanh\left(\frac{p}{4}(k_1x + k_2y - \tau_1 t)\right) + coth\left(\frac{p}{4}(k_1x + k_2y - \tau_1 t)\right)\right)\right]\right)^2 e^{i(a_1x + \tau_2 t)} \quad (16)$$

$$\nu(x,y,t)_{17} = b_0 + b_1\left(p - \frac{p}{4}\left[2 + \left(\tanh\left(\frac{p}{4}(ik_1x + k_2y - \tau_1 t)\right) + coth\left(\frac{p}{4}(ik_1x + k_2y - \tau_1 t)\right)\right)\right]\right) + b_2\left(p - \frac{p^2}{4}\left[2 + \left(\tanh\left(\frac{p}{4}(ik_1x + k_2y - \tau_1 t)\right) + coth\left(\frac{p}{4}(ik_1x + k_2y - \tau_1 t)\right)\right)\right]\right)^2 e^{i(a_1x + \tau_2 t)} \quad (17)$$

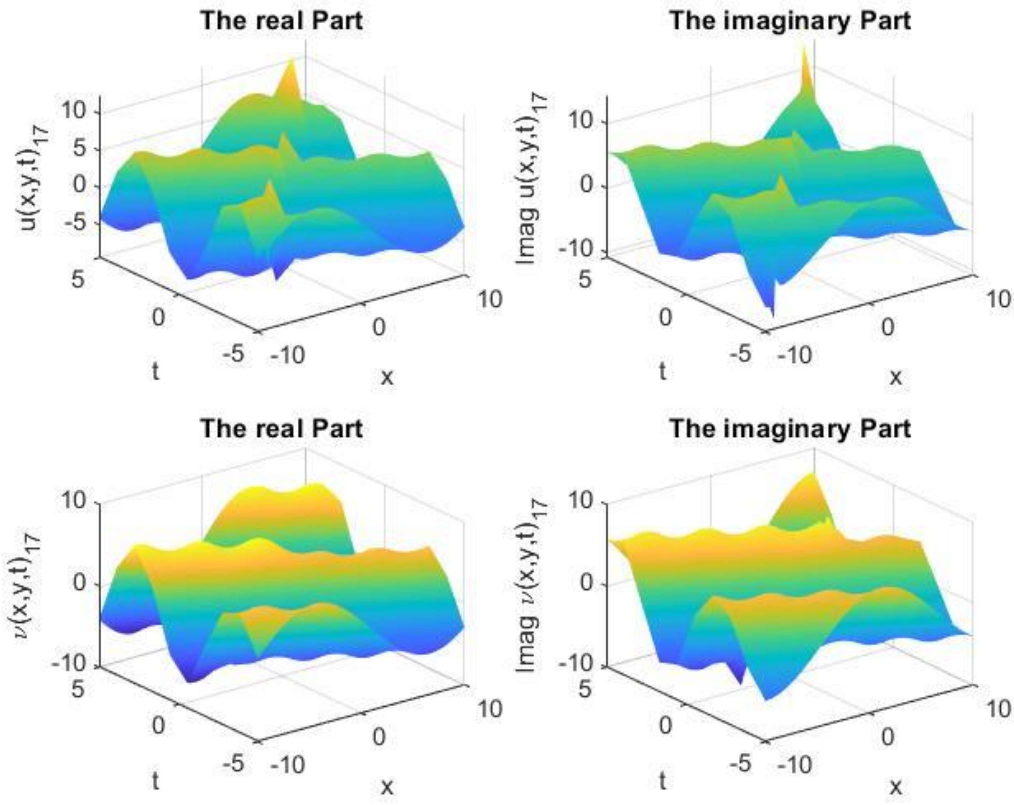

Figure 3: Plotting of the real and the Imaginary parts of $\nu_{17}(x,y,t)$ with $b_0=5$, $b_1=1$, $b_2=0.003$, $p=2$, $k_1=1.7$, $k_2=0.4$, $\tau_1=3.8$, $a_1=0.28$, $\tau_2=1$, and $x\in[-10,10]$, $y\in[-10,10]$ and $t\in[-5,5]$.

$$u(x,y,t)_{18}= b_0+b_1\left(p+\frac{p}{2}\left[-1+\frac{\sqrt{(A^2+B^2)}-A\cosh(p(k_1x+k_2y-\tau_1t))}{A\sinh(p(k_1x+k_2y-\tau_1t))+B}\right]\right) +b_2\left(p+\frac{p}{2}\left[-1+\frac{\sqrt{(A^2+B^2)}-A\cosh(p(k_1x+k_2y-\tau_1t))}{A\sinh(p(k_1x+k_2y-\tau_1t))+B}\right]\right)^2 e^{i(a_1x+\tau_2t)}, \quad (18)$$

$$v(x,y,t)_{18}= b_0+b_1\left(p+\frac{p}{2}\left[-1+\frac{\sqrt{(A^2+B^2)}-A\cosh(p(ik_1x+k_2y-\tau_1t))}{A\sinh(p(ik_1x+k_2y-\tau_1t))+B}\right]\right) +b_2\left(p+\frac{p}{2}\left[-1+\frac{\sqrt{(A^2+B^2)}-A\cosh(p(ik_1x+k_2y-\tau_1t))}{A\sinh(p(ik_1x+k_2y-\tau_1t))+B}\right]\right)^2 e^{i(a_1x+\tau_2t)}, \quad (19)$$

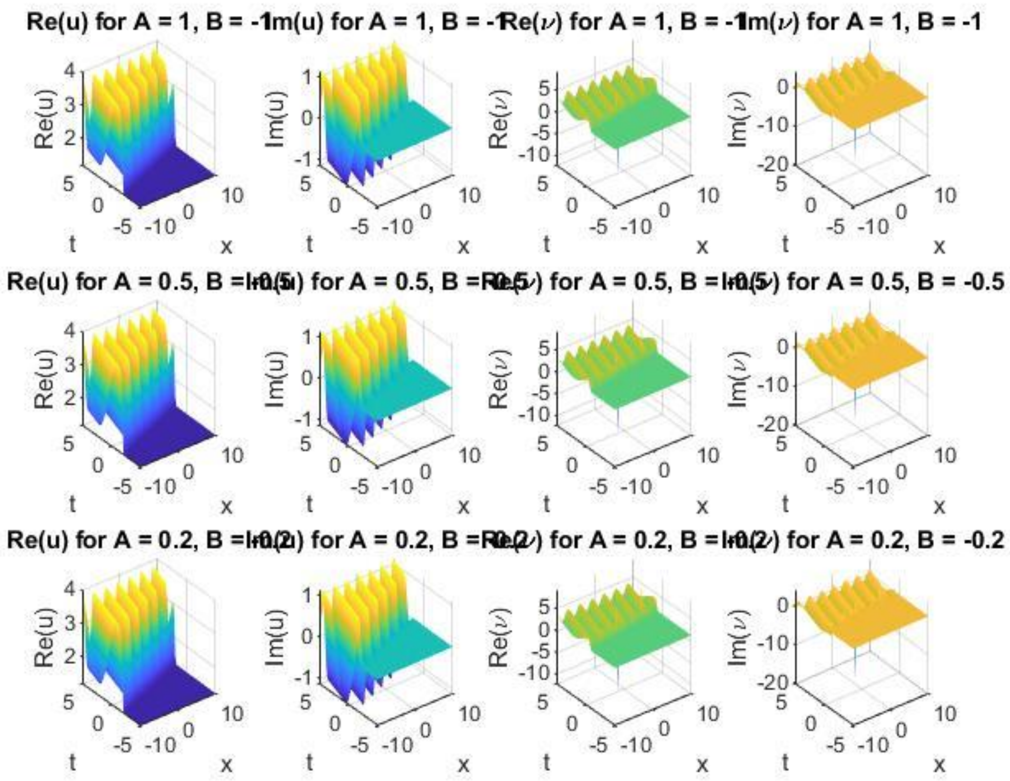

Figure 4: Plotting of the real and the Imaginary parts of $u_{18}(x,y,t)$ and $v_{18}(x,y,t)$ with $b_0 = 1.2$, $b_1 = 1$, $b_2 = 0.5$, $p = 1.5$, $k_1 = 1.2$, $k_2 = 0.8$, $\tau_1 = 10$, $a_1 = 2$, $\tau_2 = -1$, and $x \in [-10,10]$, $y \in [-10,10]$ and $t \in [0,5]$.

$$u(x,y,t)_{19} = \quad b_0 + b_1 p\left(1 + \frac{1}{2}\left[-1 - \frac{\sqrt{(B^2-A^2)} + A\cosh(p(k_1x+k_2y-\tau_1t))}{A\sinh(p(k_1x+k_2y-\tau_1t))+B}\right]\right) + b_2p^2\left(1 + \frac{1}{2}\left[-1 - \frac{\sqrt{(B^2-A^2)} + A\cosh(p(k_1x+k_2y-\tau_1t))}{A\sinh(p(k_1x+k_2y-\tau_1t))+B}\right]\right)^2 e^{i(a_1x+\tau_2t)}, \tag{20}$$

$$v(x,y,t)_{19} = \quad b_0 + b_1 p\left(1 + \frac{1}{2}\left[-1 - \frac{\sqrt{(B^2-A^2)} + A\cosh(p(ik_1x+k_2y-\tau_1t))}{A\sinh(p(ik_1x+k_2y-\tau_1t))+B}\right]\right) + b_2p^2\left(1 + \frac{1}{2}\left[-1 - \frac{\sqrt{(B^2-A^2)} + A\cosh(p(ik_1x+k_2y-\tau_1t))}{A\sinh(p(ik_1x+k_2y-\tau_1t))+B}\right]\right)^2 e^{i(a_1x+\tau_2t)}, \tag{21}$$

where A and B are two non-zero real constants and satisfies the condition $B^2 - A^2 > 0$.

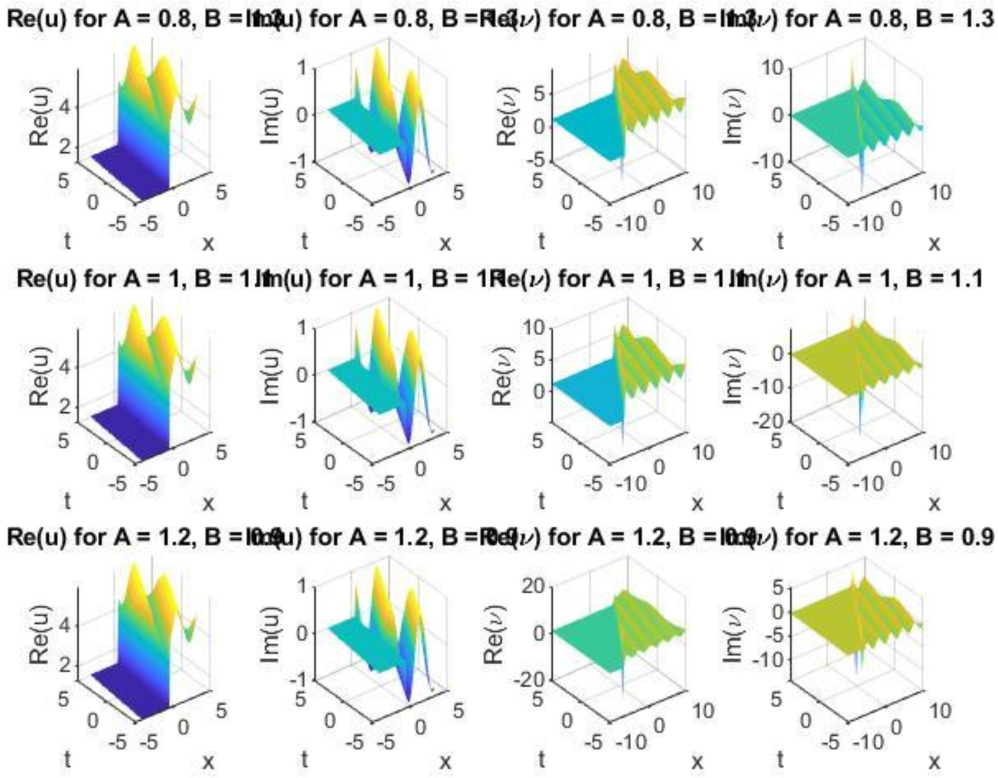

Figure 5: Plotting of the real and the Imaginary parts of $u_{19}(x,y,t)$ and $v_{19}(x,y,t)$ with $b_0 = 1.3$, $b_1 = 0.25$, $b_2 = 0.004$, $p = 15$, $k_1 = 12$, $k_2 = 0.8$, $\tau_1 = 1$, $a_1 = 2$, $\tau_2 = -1$, and $x \in [-10,10]$, $y \in [-10,10]$ and $t \in [0,5]$.

Family 3. When $r = 0$ and $pq \neq 0$, the solutions of Eq. (5) are,

$$u(x,y,t)_{25} = \quad b_0 + b_1 p\left(1 - \frac{d}{d+\cosh(p(k_1x+k_2y-\tau_1t))-\sinh(p(ik_1x+k_2y-\tau_1t))}\right) + b_2p^2\left(1 - \frac{d}{d+\cosh(p(k_1x+k_2y-\tau_1t))-\sinh(p(k_1x+k_2y-\tau_1t))}\right)^2 e^{i(a_1x+\tau_2t)}, \tag{22}$$

$$v(x,y,t)_{25} = \quad b_0 + b_1 p\left(1 - \frac{d}{d+\cosh(p(ik_1x+k_2y-\tau_1t))-\sinh(p(ik_1x+k_2y-\tau_1t))}\right) + b_2p^2\left(1 - \frac{d}{d+\cosh(p(ik_1x+k_2y-\tau_1t))-\sinh(p(ik_1x+k_2y-\tau_1t))}\right)^2 e^{i(a_1x+\tau_2t)},, \tag{23}$$

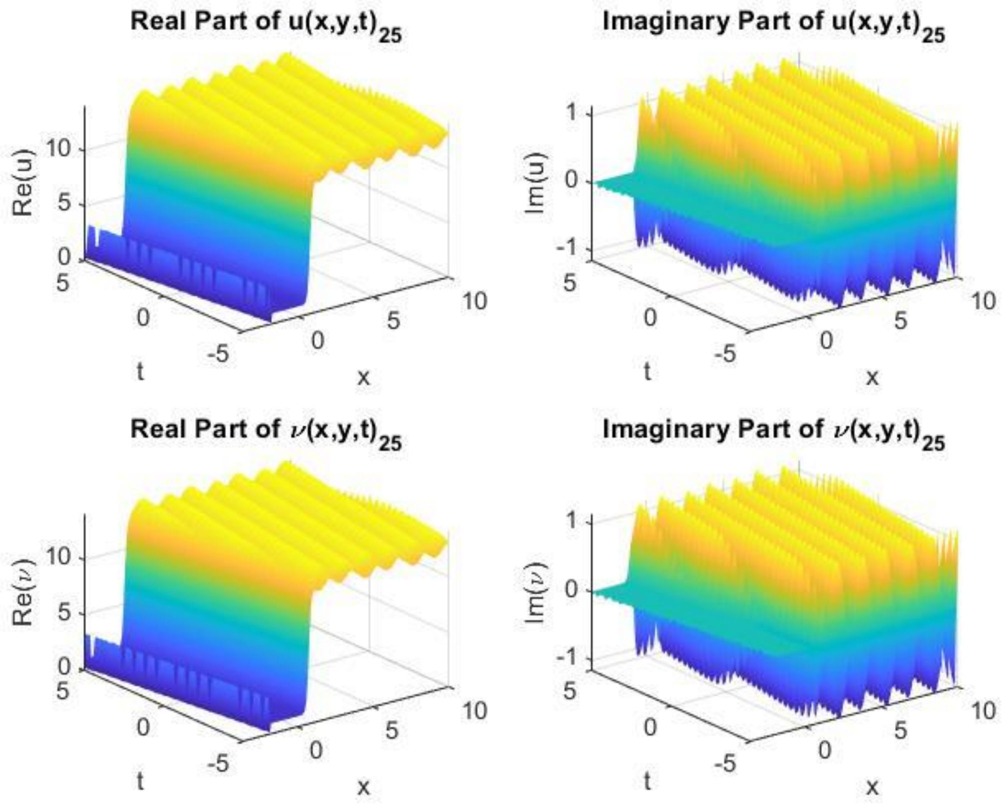

Figure6: Plotting of the real and the Imaginary parts of $u_{25}(x, y, t)$ and $\nu_{25}(x, y, t)$ with $b_0 = 1$, $b_1 = 0.5$, $b_2 = 0.8$, $p = 1.2$, $k_1 = 0.2$, $k_2 = -10.5$, $\tau_1 = 1.8$, $a_1 = 2$, $\tau_2 = 15$, $d = 1.5$, and $x \in [-10,10]$, $y \in [-10,10]$ and $t \in [0,5]$.

$$u(x,y,t)_{26} = b_0 + b_1 p\left(1 - \frac{\cosh(p(k_1x+k_2y-\tau_1t))+\sinh(p(k_1x+k_2y-\tau_1t))}{d+\cosh(p(k_1x+k_2y-\tau_1t))+\sinh(p(k_1x+k_2y-\tau_1t))}\right) + b_2 p^2\left(1 - \frac{\cosh(p(k_1x+k_2y-\tau_1t))+\sinh(p(k_1x+k_2y-\tau_1t))}{d+\cosh(p(k_1x+k_2y-\tau_1t))+\sinh(p(k_1x+k_2y-\tau_1t))}\right)^2 e^{i(a_1x+\tau_2t)}, \quad (24)$$

$$v(x,y,t)_{26} = b_0 + b_1 p\left(1 - \frac{\cosh(p(ik_1x+k_2y-\tau_1t))+\sinh(p(ik_1x+k_2y-\tau_1t))}{d+\cosh(p(ik_1x+k_2y-\tau_1t))+\sinh(p(ik_1x+k_2y-\tau_1t))}\right) + b_2 p^2\left(1 - \frac{\cosh(p(ik_1x+k_2y-\tau_1t))+\sinh(p(ik_1x+k_2y-\tau_1t))}{d+\cosh(p(ik_1x+k_2y-\tau_1t))+\sinh(p(ik_1x+k_2y-\tau_1t))}\right)^2 e^{i(a_1x+\tau_2t)}, \quad (25)$$

where d is an arbitrary constant.

Family 4. When $q \neq 0$ and $r = p = 0$, the solution of Eq. (5) is,

$$u(x,y,t)_{27} = b_1\left(\frac{-q}{q(k_1x+k_2y-\tau_1t)+d_1}\right) + b_2\left(\frac{-q}{q(k_1x+k_2y-\tau_1t)+d_1}\right)^2 e^{i(a_1x+\tau_2t)}, \quad (26)$$

$$v(x,y,t)_{27} = b_1\left(\frac{-q}{q(ik_1x+k_2y-\tau_1t)+d_1}\right) + b_2\left(\frac{-q}{q(ik_1x+k_2y-\tau_1t)+d_1}\right)^2 e^{i(a_1x+\tau_2t)}, \quad (27)$$

where d_1 is an arbitrary constant.

Case 2:

$a_j = \alpha - k_l^2\, p^2\, v$, $k_j = k_j$, $r = 0$, $p = p$, $b_2 = b_2$, $b_0 = 0$, $b_1 = -b_2\, p$, $q = q$, $\tau_1 = \tau_1$, $\tau_2 = 1$, $A = \pm\sqrt{-\frac{b_2\,\beta - 6\,k_l^2\,v}{b_2\,\gamma}}$

Family 2.

$$u(x,y,t)_{13} = b_1\left(p - \frac{p}{2}\left[1 + \tanh\left(\frac{p}{2}(k_1x + k_2y - \tau_1 t)\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \tanh\left(\frac{p}{2}(k_1x + k_2y - \tau_1 t)\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{28}$$

$$v(x,y,t)_{13} = b_1\left(p - \frac{p}{2}\left[1 + \tanh\left(\frac{p}{2}(ik_1x + k_2y - \tau_1 t)\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \tanh\left(\frac{p}{2}(ik_1x + k_2y - \tau_1 t)\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{29}$$

$$u(x,y,t)_{14} = b_1\left(p - \frac{p}{2}\left[1 + coth\left(\frac{p}{2}(k_1x + k_2y - \tau_1 t)\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + coth\left(\frac{p}{2}(k_1x + k_2y - \tau_1 t)\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{30}$$

$$v(x,y,t)_{14} = b_1\left(p - \frac{p}{2}\left[1 + coth\left(\frac{p}{2}(ik_1x + k_2y - \tau_1 t)\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + coth\left(\frac{p}{2}(ik_1x + k_2y - \tau_1 t)\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{31}$$

$$u(x,y,t)_{15} = b_1\left(p - \frac{p}{2}\left[1 + \left(\tanh(p(k_1x + k_2y - \tau_1 t)) \pm isech(p(k_1x + k_2y - \tau_1 t))\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \left(\tanh(p(k_1x + k_2y - \tau_1 t)) \pm isech(p(k_1x + k_2y - \tau_1 t))\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{32}$$

$$v(x,y,t)_{15} = b_1\left(p - \frac{p}{2}\left[1 + \left(\tanh(p(ik_1x + k_2y - \tau_1 t)) \pm isech(p(ik_1x + k_2y - \tau_1 t))\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \left(\tanh(p(ik_1x + k_2y - \tau_1 t)) \pm isech(p(ik_1x + k_2y - \tau_1 t))\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{33}$$

$$u(x,y,t)_{16} = b_1\left(p - \frac{p}{2}\left[1 + \left(coth(p(k_1x + k_2y - \tau_1 t)) \pm csch(p(k_1x + k_2y - \tau_1 t))\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \left((coth(p(k_1x + k_2y - \tau_1 t)) \pm csch(p(k_1x + k_2y - \tau_1 t))\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{34}$$

$$v(x,y,t)_{16} = b_1\left(p - \frac{p}{2}\left[1 + \left((coth(p(ik_1x + k_2y - \tau_1 t)) \pm csch(p(ik_1x + k_2y - \tau_1 t))\right)\right]\right) + b_2\left(p - \frac{p^2}{2}\left[1 + \left((coth(p(ik_1x + k_2y - \tau_1 t)) \pm csch(p(ik_1x + k_2y - \tau_1 t))\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{35}$$

$$u(x,y,t)_{17} = b_1\left(p - \frac{p}{4}\left[2 + \left(\tanh\left(\frac{p}{4}(k_1x + k_2y - \tau_1 t)\right) + coth\left(\frac{p}{4}(k_1x + k_2y - \tau_1 t)\right)\right)\right]\right) + b_2\left(p - \frac{p^2}{4}\left[2 + \left(\tanh\left(\frac{p}{4}(k_1x + k_2y - \tau_1 t)\right) + coth\left(\frac{p}{4}(k_1x + k_2y - \tau_1 t)\right)\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{36}$$

$$v(x,y,t)_{17} = b_1\left(p - \frac{p}{4}\left[2 + \left(\tanh\left(\frac{p}{4}(ik_1x + k_2y - \tau_1 t)\right) + coth\left(\frac{p}{4}(ik_1x + k_2y - \tau_1 t)\right)\right)\right]\right) + b_2\left(p - \frac{p^2}{4}\left[2 + \left(\tanh\left(\frac{p}{4}(ik_1x + k_2y - \tau_1 t)\right) + coth\left(\frac{p}{4}(ik_1x + k_2y - \tau_1 t)\right)\right)\right]\right)^2 e^{i(a_1x+\tau_2 t)} \tag{37}$$

$$u(x,y,t)_{18} = b_1\left(p + \frac{p}{2}\left[-1 + \frac{\sqrt{(A^2+B^2)} - A\cosh(p(k_1x+k_2y-\tau_1 t))}{A\sinh(p(k_1x+k_2y-\tau_1 t)) + B}\right]\right) + b_2\left(p + \frac{p}{2}\left[-1 + \frac{\sqrt{(A^2+B^2)} - A\cosh(p(k_1x+k_2y-\tau_1 t))}{A\sinh(p(k_1x+k_2y-\tau_1 t)) + B}\right]\right)^2 e^{i(a_1x+\tau_2 t)}, \tag{38}$$

$$v(x,y,t)_{18} = b_1\left(p + \frac{p}{2}\left[-1 + \frac{\sqrt{(A^2+B^2)} - A\cosh(p(ik_1x+k_2y-\tau_1 t))}{A\sinh(p(ik_1x+k_2y-\tau_1 t)) + B}\right]\right) + b_2\left(p + \frac{p}{2}\left[-1 + \frac{\sqrt{(A^2+B^2)} - A\cosh(p(ik_1x+k_2y-\tau_1 t))}{A\sinh(p(ik_1x+k_2y-\tau_1 t)) + B}\right]\right)^2 e^{i(a_1x+\tau_2 t)}, \tag{39}$$

$$u(x,y,t)_{19} = b_1 p\left(1+\frac{1}{2}\left[-1-\frac{\sqrt{(B^2-A^2)}+A\cosh(p(k_1x+k_2y-\tau_1 t))}{A\sinh(p(k_1x+k_2y-\tau_1 t))+B}\right]\right) + b_2 p^2\left(1+\frac{1}{2}\left[-1-\frac{\sqrt{(B^2-A^2)}+A\cosh(p(k_1x+k_2y-\tau_1 t))}{A\sinh(p(k_1x+k_2y-\tau_1 t))+B}\right]\right)^2 e^{i(a_1x+\tau_2 t)}, \quad (40)$$

$$v(x,y,t)_{19} = b_1 p\left(1+\frac{1}{2}\left[-1-\frac{\sqrt{(B^2-A^2)}+A\cosh(p(ik_1x+k_2y-\tau_1 t))}{A\sinh(p(ik_1x+k_2y-\tau_1 t))+B}\right]\right) + b_2 p^2\left(1+\frac{1}{2}\left[-1-\frac{\sqrt{(B^2-A^2)}+A\cosh(p(ik_1x+k_2y-\tau_1 t))}{A\sinh(p(ik_1x+k_2y-\tau_1 t))+B}\right]\right)^2 e^{i(a_1x+\tau_2 t)}, \quad (41)$$

where A and B are two non-zero real constants and satisfies the condition $B^2 - A^2 > 0$.

Family 3. When $r = 0$ and $pq \neq 0$, the solutions of Eq. (5) are,

$$u(x,y,t)_{25} = b_1 p\left(1-\frac{d}{d+\cosh(p(k_1x+k_2y-\tau_1 t))-\sinh(p(ik_1x+k_2y-\tau_1 t))}\right) + b_2 p^2\left(1-\frac{d}{d+\cosh(p(k_1x+k_2y-\tau_1 t))-\sinh(p(k_1x+k_2y-\tau_1 t))}\right)^2 e^{i(a_1x+\tau_2 t)}, \quad (42)$$

$$v(x,y,t)_{25} = b_1 p\left(1-\frac{d}{d+\cosh(p(ik_1x+k_2y-\tau_1 t))-\sinh(p(ik_1x+k_2y-\tau_1 t))}\right) + b_2 p^2\left(1-\frac{d}{d+\cosh(p(ik_1x+k_2y-\tau_1 t))-\sinh(p(ik_1x+k_2y-\tau_1 t))}\right)^2 e^{i(a_1x+\tau_2 t)},, \quad (43)$$

$$u(x,y,t)_{26} = b_1 p\left(1-\frac{\cosh(p(k_1x+k_2y-\tau_1 t))+\sinh(p(k_1x+k_2y-\tau_1 t))}{d+\cosh(p(k_1x+k_2y-\tau_1 t))+\sinh(p(k_1x+k_2y-\tau_1 t))}\right) + b_2 p^2\left(1-\frac{\cosh(p(k_1x+k_2y-\tau_1 t))+\sinh(p(k_1x+k_2y-\tau_1 t))}{d+\cosh(p(k_1x+k_2y-\tau_1 t))+\sinh(p(k_1x+k_2y-\tau_1 t))}\right)^2 e^{i(a_1x+\tau_2 t)}, \quad (44)$$

$$v(x,y,t)_{26} = b_1 p\left(1-\frac{\cosh(p(ik_1x+k_2y-\tau_1 t))+\sinh(p(ik_1x+k_2y-\tau_1 t))}{d+\cosh(p(ik_1x+k_2y-\tau_1 t))+\sinh(p(ik_1x+k_2y-\tau_1 t))}\right) + b_2 p^2\left(1-\frac{\cosh(p(ik_1x+k_2y-\tau_1 t))+\sinh(p(ik_1x+k_2y-\tau_1 t))}{d+\cosh(p(ik_1x+k_2y-\tau_1 t))+\sinh(p(ik_1x+k_2y-\tau_1 t))}\right)^2 e^{i(a_1x+\tau_2 t)}, \quad (45)$$

where d is an arbitrary constant.

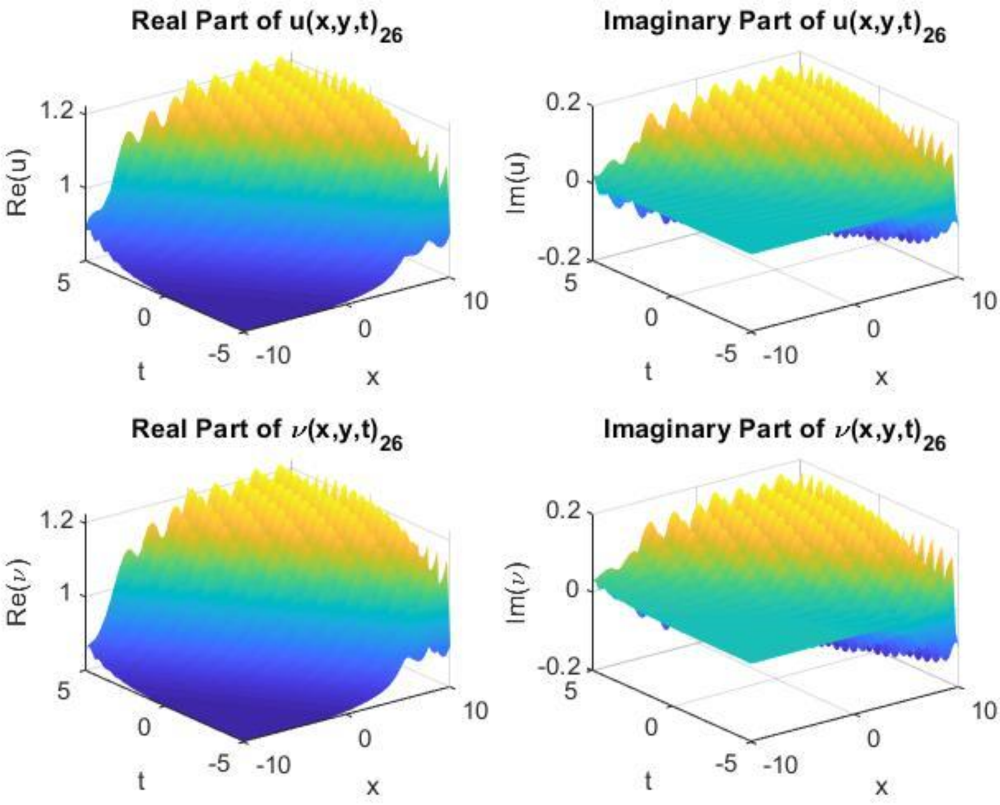

Figure 7: Plotting of the real and the Imaginary parts of $u_{26}(x,y,t)$ and $v_{26}(x,y,t)$ with $b_0 = 0.8$, $b_1 = 0.5$, $b_2 = 0.7$, $p = 50.3$, $k_1 = 0.1$, $k_2 = -1$, $\tau_1 = 1.5$, $a_1 = 1$, $\tau_2 = 10$, $d = 1.5$, and $x \in [-10,10]$, $y \in [-10,10]$ and $t \in [0,5]$.

Family 4. When $q \neq 0$ and $r = p = 0$, the solution of Eq. (5) is,

$$u(x,y,t)_{27} = \quad b_1\left(\frac{-q}{q(k_1x+k_2y-\tau_1t)+d_1}\right) + b_2\left(\frac{-q}{q(k_1x+k_2y-\tau_1t)+d_1}\right)^2 e^{i(a_1x+\tau_2t)}, \tag{46}$$

$$v(x,y,t)_{27} = \quad b_1\left(\frac{-q}{q(ik_1x+k_2y-\tau_1t)+d_1}\right) + b_2\left(\frac{-q}{q(ik_1x+k_2y-\tau_1t)+d_1}\right)^2 e^{i(a_1x+\tau_2t)}, \tag{47}$$

where d_1 is an arbitrary constant.

Case 3:

$$a_j = \alpha - 16\,k_l^2\,q\,r\,v, \quad k_j = k_j, \quad r = r, \quad p = 0, \quad b_2 = b_2, \quad b_0 = -\frac{8\,b_2\,q\,r}{3}, \quad b_1 = 0,$$

$$q = q, \quad \tau_1 = \tau_1, \quad \tau_2 = 1, \quad A = \pm\sqrt{-\frac{b_2\,\beta - 6\,k_l^2\,v}{b_2\,\gamma}}$$

For $q\,r\, > 0$

Family 1.

$$u(x,y,t)_1 = \quad b_0 + b_1\left(\frac{qr}{\sqrt{qr}\tan(\sqrt{qr}(k_1x+k_2y-\tau_1t))} + \sqrt{qr}\tan\left(\sqrt{qr}(k_1x + k_2y - \tau_1t)\right)\right)$$
$$+b_2\left(\frac{qr}{\sqrt{qr}\tan(\sqrt{qr}(k_1x+k_2y-\tau_1t))} + 2\sqrt{qr}\tan\left(\sqrt{qr}(k_1x + k_2y - \tau_1t)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \tag{48}$$

$$v(x,y,t)_1 = b_0 + b_1\left(\frac{qr}{\sqrt{qr}\tan(\sqrt{qr}(ik_1x+k_2y-\tau_1t))} + \sqrt{qr}\tan\left(\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right) + b_2\left(\frac{qr}{\sqrt{qr}\tan(\sqrt{qr}(ik_1x+k_2y-\tau_1t))} + 2\sqrt{qr}\tan\left(\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \tag{49}$$

$$u(x,y,t)_2 = b_0 + b_1\left(\frac{qr}{\sqrt{qr}\cot(\sqrt{qr}(k_1x+k_2y-\tau_1t))} + \sqrt{qr}\cot\left(\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right) + b_2\left(\frac{qr}{\sqrt{qr}\cot(\sqrt{qr}(k_1x+k_2y-\tau_1t))} + 2\sqrt{qr}\cot\left(\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \tag{50}$$

$$v(x,y,t)_2 = b_0 + b_1\left(\frac{qr}{\sqrt{qr}\cot(\sqrt{qr}(ik_1x+k_2y-\tau_1t))} + \sqrt{qr}\cot\left(\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right) + b_2\left(\frac{qr}{\sqrt{qr}\cot(\sqrt{qr}(ik_1x+k_2y-\tau_1t))} + 2\sqrt{qr}\cot\left(\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \tag{51}$$

$$u(x,y,t)_3 = b_0 + b_1\left(\frac{qr}{\sqrt{qr}\left(\tan(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm\sec(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\tan\left(\sqrt{4qr}(k_1x+k_2y-\tau_1t)\right)\pm\sec\left(\sqrt{4qr}(k_1x+k_2y-\tau_1t)\right)\right)\right) + b_2\left(\frac{qr}{\sqrt{qr}\left(\tan(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm\sec(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\tan\left(\sqrt{4qr}(k_1x+k_2y-\tau_1t)\right)\pm\sec\left(\sqrt{4qr}(k_1x+k_2y-\tau_1t)\right)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \tag{52}$$

$$v(x,y,t)_3 = b_0 + b_1\left(\frac{qr}{\sqrt{qr}\left(\tan(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm\sec(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\tan\left(\sqrt{4qr}(ik_1x+k_2y-\tau_1t)\right)\pm\sec\left(\sqrt{4qr}(ik_1x+k_2y-\tau_1t)\right)\right)\right) + b_2\left(\frac{qr}{\sqrt{qr}\left(\tan(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm\sec(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\tan\left(\sqrt{4qr}(ik_1x+k_2y-\tau_1t)\right)\pm\sec\left(\sqrt{4qr}(ik_1x+k_2y-\tau_1t)\right)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \tag{53}$$

$$u(x,y,t)_4 = b_0 + b_1\left(\frac{-qr}{\sqrt{qr}\left(\cot(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm\csc(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\right)} - \sqrt{qr}\left(\cot\left(\sqrt{4qr}(k_1x+k_2y-\tau_1t)\right)\pm\csc\left(\sqrt{4qr}(k_1x+k_2y-\tau_1t)\right)\right)\right) + b_2\left(\frac{-qr}{\sqrt{qr}\left(\cot(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm\csc(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\right)} - \sqrt{qr}\left(\cot\left(\sqrt{4qr}(k_1x+k_2y-\tau_1t)\right)\pm\csc\left(\sqrt{4qr}(k_1x+k_2y-\tau_1t)\right)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \tag{54}$$

$$v(x,y,t)_4 = b_0 + b_1\left(\frac{-qr}{\sqrt{qr}\left(\cot(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm\csc(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\right)} - \sqrt{qr}\left(\cot\left(\sqrt{4qr}(ik_1x+k_2y-\tau_1t)\right)\pm\csc\left(\sqrt{4qr}(ik_1x+k_2y-\tau_1t)\right)\right)\right) + b_2\left(\frac{-qr}{\sqrt{qr}\left(\cot(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm\csc(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\right)} - \sqrt{qr}\left(\cot\left(\sqrt{4qr}(ik_1x+k_2y-\tau_1t)\right)\pm\csc\left(\sqrt{4qr}(ik_1x+k_2y-\tau_1t)\right)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (55)$$

$$u(x,y,t)_5 = b_0 + b_1\left(\frac{2qr}{\sqrt{qr}\left(\tan\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)-\cot\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right)} + \frac{1}{2}\sqrt{qr}\left(\tan\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)-\cot\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right)\right) + b_2\left(\frac{2qr}{\sqrt{qr}\left(\tan\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)-\cot\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right)} + \frac{1}{2}\sqrt{qr}\left(\tan\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)-\cot\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (56)$$

$$v(x,y,t)_5 = b_0 + b_1\left(\frac{2qr}{\sqrt{qr}\left(\tan\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)-\cot\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right)} + \frac{1}{2}\sqrt{qr}\left(\tan\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)-\cot\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right)\right) + b_2\left(\frac{2qr}{\sqrt{qr}\left(\tan\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)-\cot\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right)} e^{i(a_1x+\tau_2t)}, \quad (57)$$

$$u(x,y,t)_6 = b_0 + b_1\left(\frac{qr(A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B)}{\sqrt{qr}\left(\sqrt{(A^2-B^2)}-A\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\frac{\sqrt{(A^2-B^2)}-A\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))}{A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B}\right)\right) + b_2\left(\frac{qr(A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B)}{\sqrt{qr}\left(\sqrt{(A^2-B^2)}-A\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\frac{\sqrt{(A^2-B^2)}-A\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))}{A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B}\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (58)$$

$$v(x,y,t)_6 = b_0 + b_1\left(\frac{qr(A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B)}{\sqrt{qr}\left(\sqrt{(A^2-B^2)}-A\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\frac{\sqrt{(A^2-B^2)}-A\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))}{A\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))+B}\right)\right) + b_2\left(\frac{qr(A\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))+B)}{\sqrt{qr}\left(\sqrt{(A^2-B^2)}-A\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\frac{\sqrt{(A^2-B^2)}-A\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))}{A\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))+B}\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (59)$$

$$u(x,y,t)_7 = b_0 + b_1\left(\frac{qr(A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B)}{\sqrt{qr}\left(\sqrt{(A^2-B^2)}+A\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\frac{\sqrt{(A^2-B^2)}+A\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))}{A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B}\right)\right) + b_2\left(\frac{qr(A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B)}{\sqrt{qr}\left(\sqrt{(A^2-B^2)}+A\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\frac{\sqrt{(A^2-B^2)}+A\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))}{A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B}\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (60)$$

$$v(x,y,t)_7 = b_0 + b_1\left(\frac{qr(A\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))+B)}{\sqrt{qr}\left(\sqrt{(A^2-B^2)}+A\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\frac{\sqrt{(A^2-B^2)}+A\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))}{A\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))+B}\right)\right) + b_2\left(\frac{qr(A\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))+B)}{\sqrt{qr}\left(\sqrt{(A^2-B^2)}+A\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\right)} + \sqrt{qr}\left(\frac{\sqrt{(A^2-B^2)}+A\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))}{A\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))+B}\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (61)$$

where A and B are two non-zero real constants and satisfies the condition $A^2 - B^2 > 0$.

$$u(x,y,t)_8 = b_0 + b_1\left(\frac{-\sqrt{qr}}{\cot(\sqrt{qr}(k_1x+k_2y-\tau_1t))} - \sqrt{qr}\cot\left(\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right) + b_2\left(\frac{-\sqrt{qr}}{\cot(\sqrt{qr}(k_1x+k_2y-\tau_1t))} - \sqrt{qr}\cot\left(\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (62)$$

$$v(x,y,t)_8 = b_0 + b_1\left(\frac{-\sqrt{qr}}{\cot(\sqrt{qr}(ik_1x+k_2y-\tau_1t))} - \sqrt{qr}\cot\left(\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right) + b_2\left(\frac{-\sqrt{qr}}{\cot(\sqrt{qr}(ik_1x+k_2y-\tau_1t))} - \sqrt{qr}\cot\left(\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (63)$$

$$u(x,y,t)_9 = b_0 + b_1\left(\frac{\sqrt{qr}}{\tan(\sqrt{qr}(k_1x+k_2y-\tau_1t))} + \sqrt{qr}\tan\left(\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right) + b_2\left(\frac{\sqrt{qr}}{\tan(\sqrt{qr}(k_1x+k_2y-\tau_1t))} + \sqrt{qr}\tan\left(\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (64)$$

$$v(x,y,t)_9 = b_0 + b_1\left(\frac{\sqrt{qr}}{\tan(\sqrt{qr}(ik_1x+k_2y-\tau_1t))} + \sqrt{qr}\tan\left(\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right) + b_2\left(\frac{\sqrt{qr}}{\tan(\sqrt{qr}(ik_1x+k_2y-\tau_1t))} + \sqrt{qr}\tan\left(\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\right)^2 e^{i(a_1x+\tau_2t)}, \quad (65)$$

$$u(x,y,t)_{10} = b_0 + b_1\left(\frac{-\sqrt{qr}(\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm1)}{\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))} - \frac{\sqrt{qr}\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))}{(\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm1)}\right) + b_2\left(\frac{-\sqrt{qr}(\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm1)}{\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))} - \frac{\sqrt{qr}\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))}{(\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm1)}\right)^2 e^{i(a_1x+\tau_2t)}, \quad (66)$$

$$v(x,y,t)_{10} = b_0 + b_1\left(\frac{-\sqrt{qr}(\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm1)}{\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))} - \frac{\sqrt{qr}\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))}{(\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm1)}\right) + b_2\left(\frac{-\sqrt{qr}(\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm1)}{\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))} - \frac{\sqrt{qr}\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))}{(\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm1)}\right)^2 e^{i(a_1x+\tau_2t)}, \tag{67}$$

$$u(x,y,t)_{11} = b_0 + b_1\left(\frac{\sqrt{qr}(\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm1)}{\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))} + \frac{\sqrt{qr}\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))}{(\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm1)}\right) + b_2\left(\frac{\sqrt{qr}(\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm1)}{\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))} + \frac{\sqrt{qr}\sin(\sqrt{4qr}(k_1x+k_2y-\tau_1t))}{(\cos(\sqrt{4qr}(k_1x+k_2y-\tau_1t))\pm1)}\right)^2 e^{i(a_1x+\tau_2t)}, \tag{68}$$

$$v(x,y,t)_{11} = b_0 + b_1\left(\frac{\sqrt{qr}(\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm1)}{\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))} + \frac{\sqrt{qr}\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))}{(\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm1)}\right) + b_2\left(\frac{\sqrt{qr}(\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm1)}{\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))} + \frac{\sqrt{qr}\sin(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))}{(\cos(\sqrt{4qr}(ik_1x+k_2y-\tau_1t))\pm1)}\right)^2 e^{i(a_1x+\tau_2t)}, \tag{69}$$

$$u(x,y,t)_{12} = b_0 + b_1\left(\frac{\sqrt{qr}\left(2\cos^2\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)-1\right)}{2\sin\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\cos\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)} + \frac{2\sqrt{qr}\sin\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\cos\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)}{\left(2\cos^2\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)-1\right)}\right) + b_2\left(\frac{\sqrt{qr}\left(2\cos^2\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)-1\right)}{2\sin\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\cos\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)} + \frac{2\sqrt{qr}\sin\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)\cos\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)}{\left(2\cos^2\left(\frac{1}{2}\sqrt{qr}(k_1x+k_2y-\tau_1t)\right)-1\right)}\right)^2 e^{i(a_1x+\tau_2t)}, \tag{70}$$

$$v(x,y,t)_{12} = b_0 + b_1\left(\frac{\sqrt{qr}\left(2\cos^2\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)-1\right)}{2\sin\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\cos\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)} + \frac{2\sqrt{qr}\sin\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\cos\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)}{\left(2\cos^2\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)-1\right)}\right) + b_2\left(\frac{\sqrt{qr}\left(2\cos^2\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)-1\right)}{2\sin\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\cos\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)} + \frac{2\sqrt{qr}\sin\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)\cos\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)}{\left(2\cos^2\left(\frac{1}{2}\sqrt{qr}(ik_1x+k_2y-\tau_1t)\right)-1\right)}\right)^2 e^{i(a_1x+\tau_2t)}, \tag{71}$$

4. Conclusion

The analytical solutions derived using the (G'/G)-expansion method offer valuable insights into the complex dynamics of pulse interactions in optical fibers. The wave profiles elucidate the critical roles of self- and cross-phase modulation in shaping pulse propagation. SPM primarily leads to pulse broadening and frequency chirping, while XPM introduces inter-pulse coupling that can significantly affect the stability of optical signals. These findings are essential for improving the design and performance of fiber-optic communication systems, where precise control over pulse propagation is crucial for minimizing signal distortion and maintaining data integrity. Overall, the results not only deepen our understanding of nonlinear optical phenomena but also contribute to the advancement of technologies in high-speed optical communication and pulse shaping.

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